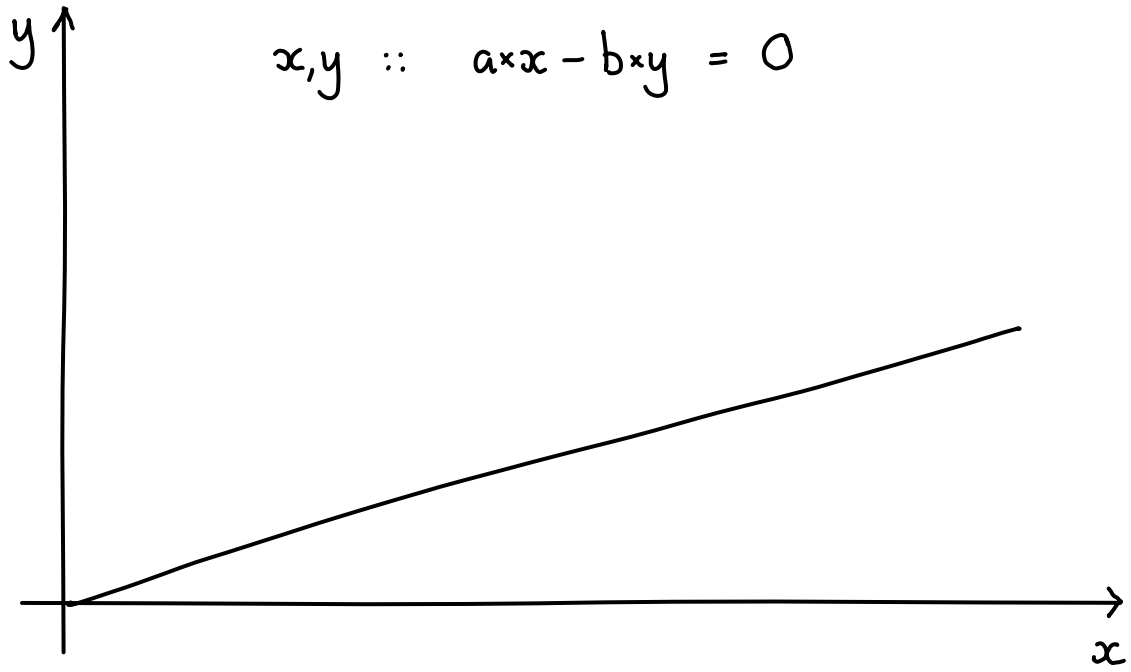
Raster-Display Algorithms

Note Title 25/10/2006



Integer closest to x : $\lceil x - \frac{1}{2} \rceil$

Approximate the straight line by a function g from integers to integers given by

$$g.m = \left\lceil \frac{a \times m}{b} - \frac{1}{2} \right\rceil$$

Plotting algorithm:

```
{true}
m,n := 0,0;
{Invariant:  n = g.m}
do  true →  plot(m,n) ;
            m,n := m+1, g.(m+1)
od.
```

Assume $0 \leqslant a \leqslant b \;\wedge\; 0 < b$

$\{ 0 \leqslant a \leqslant b \;\wedge\; 0 < b \}$
$m,n := 0,0;$
$\{ \text{Invariant: } n = g.m \}$
$\textbf{do}\ \text{true} \longrightarrow\ \text{plot}(m,n)\ ;$

$\qquad\qquad m,n := m+1, g.(m+1)$

$\textbf{od}.$

The Ceiling Function

For all reals x and integers m

$$\lceil x \rceil \leqslant m \;\equiv\; x \leqslant m \quad .$$

Contrapositive:

$$\neg(\lceil x \rceil \leqslant m) \;\equiv\; \neg(x \leqslant m)$$

I.e. $\quad m < \lceil x \rceil \;\equiv\; m < x \quad .$